UDC 621.396.946
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621.3.029.64

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# SATELLITE BROADCASTING: a zoned reflector aerial for the domestic reception of Band VI

## No. 1972/10

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

RESEARCH DEPARTMENT

# SATELLITE BROADCASTING: A ZONED REFLECTOR AERIAL FOR THE DOMESTIC RECEPTION OF BAND VI

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(RA-96)

Research Department Report No. 1972/10

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# SATELLITE BROADCASTING: A ZONED REFLECTOR AERIAL FOR THE DOMESTIC RECEPTION OF BAND VI

## Summary

*The design and measured performance of a new type of aerial which might be suitable for the domestic reception of Band VI (11·7 – 12·7 GHz) signals from a geo-stationary satellite are described. It is believed that in sufficient quantity it could be manufactured fairly cheaply.*

## 1. Introduction

The use of Band VI (11·7 – 12·7 GHz) has been considered for broadcasting television signals from a geo-stationary satellite. If direct domestic reception were employed the receiving aerial would require a high gain to provide adequate signal level at the receiver input and low sidelobe levels to minimise interference from neighbouring satellites. Both of the requirements could be satisfied by a conventional type of aerial employing a suitably illuminated parabolic dish. Considerations of cost coupled with the difficulty of mounting a large parabolic dish to ensure sufficient stability of beam direction under windy conditions suggested the investigation of other forms of aerials.

This report describes a receiving aerial which employs a stepped reflector in place of the paraboloid. It is believed that this aerial could offer a number of advantages over the conventional type of which the most important are probably:—

(a) less windage and easier mounting,

(b) the possibility of incorporating a fixed beam slew into the design and

(c) the availability of cheap fabricating techniques.

## 2. Aerial requirements

The following tentative aerial parameters, derived from the performance of a parabolic reflector, have been proposed[1] as representative of those required for a domestic aerial receiving Band VI signals from a geo-stationary satellite.

| | |
|---|---|
| Aerial diameter: | 75 cms |
| Mean wavelength: | 2·5 cms |
| 3 dB beamwidth: | ±1·25° |
| Bandwidth: | ±400 MHz |
| Envelope of off-axis response: | $-(9 + 20 \log \phi/\phi_0)$ dB |

(1)

where $\phi$ is the angle from the beam axis and $\phi_0$ the total 3 dB beamwidth. For interference calculations a minimum response of –30 dB is assumed.

The narrow beamwidth requires that the aerial be rigidly mounted to obviate signal variation under windy conditions and easily steerable for lining up with the satellite transmitter. In the United Kingdom the angle of elevation of a geo-stationary satellite located due south varies from 24° to 33° and for a given satellite position this angle varies only slowly for small changes of longitude.

## 3. Description of aerial

### 3.1. Geometry

The aerial consists of a horn feed and a zoned reflector (Fig. 1). The reflector (Fig. 2) is a surface of revolution about the common axis of a series of confocal parabolae ($L_1$, $L_2$, $L_3$ etc.) the directrices of which are separated by one wavelength (3·2 cm) at the design frequency of 9375 MHz.* XOX′ is parallel to the tangent plane at the vertex V of $L_1$ and lines drawn from the common focus F to the intersections of XOX′ and $L_1$ are continued to meet $L_2$.

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* The choice of this frequency was dictated by the availability of measuring equipment.

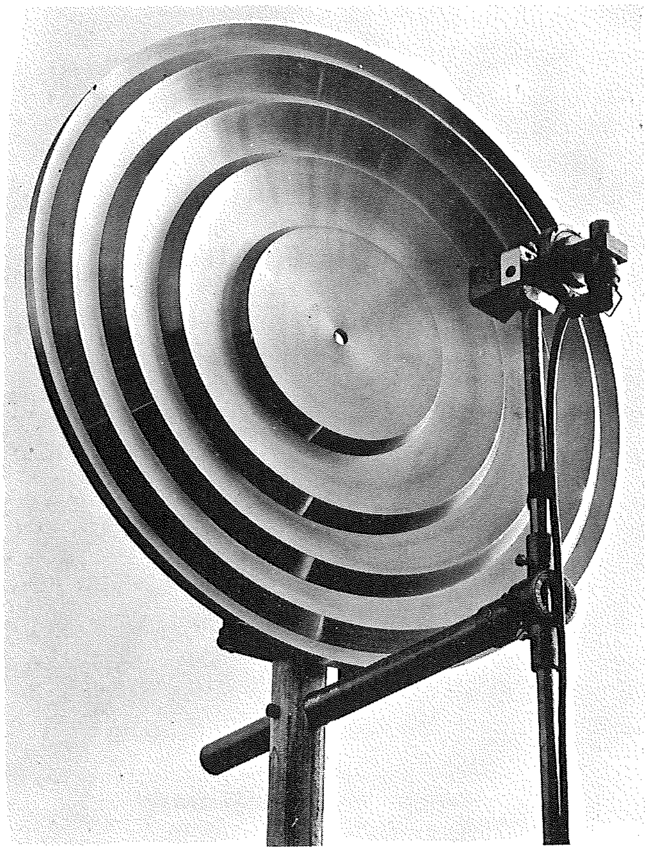

*Fig. 1 - Experimental zoned reflector aerial*

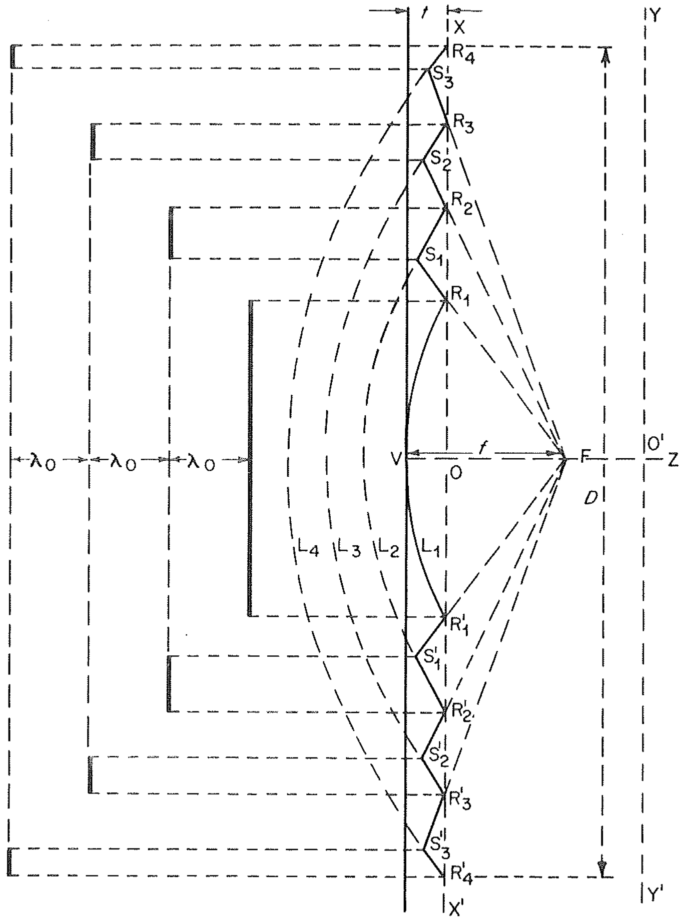

*Fig. 2 - Development of zoned reflector*

This process is continued for the intersections of XOX' and $L_2$, $L_3$ etc. The focal distance VF ($=f$) is 0·37 times the reflector diameter D and the thickness $t$ was arbitrarily chosen to make a conveniently shallow reflector.

To ease manufacturing problems the centre part of the reflector and the zones were made in spherical form. The radii of the spheres (the centres of which lie on FZ) were adjusted to make spheres and paraboloids coincide at $R_1$, V, $R_1'$; $R_2$, $S_1$, $S_1'$, $R_2'$; $R_3$, $S_2$, $S_2'$, $R_3'$ etc.

The error in making the sections spherical instead of paraboloidal is of the order of $t^2/8\rho$ where $\rho$ is the radius of the appropriate spherical section. The minimum value of $\rho$ is $2f + t/2$ corresponding to a maximum error $<0{\cdot}01$ cm.

The availability of machines restricted the overall diameter of the reflector to 54 cms. The thickness $t$ was chosen to be 1 cm which resulted in four full and one partial zone (see Fig. 1).

The master reflector shown in Fig. 1 was machined from light alloy. A glass-fibre mould was also made and copies of the reflector produced using silver plated nylon mesh supported on a glass-fibre structure.

### 3.2. Mode of operation

On the basis of an optical analogy rays originating at the focus of a parabola, after reflection from the surface, appear to originate on the directrix, hence in the case of the zoned reflector the radiation appears to be emitted from a central circular area and a number of annular regions spaced $\lambda_0$, $2\lambda_0$, $3\lambda_0$ etc. behind the central area – $\lambda_0$ being the design wavelength (see Fig. 2).

The illumination of the aperture in a plane through YO'Y' normal to the axis is co-phased at the design frequency but as the zones are not contiguous the gain, which is proportional to the illuminated area, will be reduced by a factor

$$\frac{4 \text{ x total illuminated area}}{\pi D^2}$$

In the case of the reflector of Fig. 1 this factor is 0·75 corresponding to a gain of 1·25 dB less than that of a paraboloid of the same diameter and illumination. The spacing of the zones necessarily reduces the bandwidth vis-à-vis that of a single paraboloid but with the small percentage bandwidth required this would not be a serious disadvantage.

The ratio $f/D = 0{\cdot}37$ was selected to give ±4 beamwidths beam scan for not more than 1 dB loss of gain by moving the horn feed and was based on the assumption that the performance would not differ substantially from that given by Silver[2] for a full paraboloid.

Increasing the $f/D$ ratio above the 0·25 of a focal plane reflector has the further advantages that fewer zones are required for a given $t$ and the beam deviation factor is greater[3] at the expense of putting the feed further in front of the reflector.

The feed horn was designed to give a 10 dB taper from the centre to the edge of the dish.

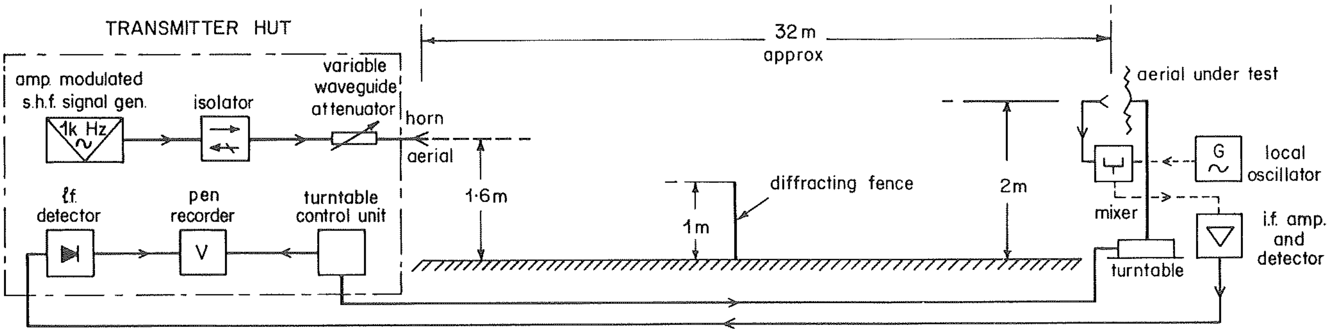

*Fig. 3 - Measuring site arrangement.*

## 4. Measurement procedure

The minimum distance for far-field measurements can be taken as 18·3 m ($2D^2/\lambda$) and the distance used was 32 m. To minimise errors due to ground reflections a diffracting fence (see Appendix I) was employed. It was particularly important to minimise ground reflections since the side lobes, being annular in form, radiated almost as strongly towards the ground as in the direction of the receiver unlike the main beam which radiated little towards the ground, and without the fence large errors in relative side-lobe levels could occur.

Fig. 3 shows the measuring arrangement. The output of an s.h.f. signal generator amplitude modulated at 1 kHz was fed via an isolator and variable attenuator to a low gain horn. The received signal was mixed with a local oscillator signal and passed through a 8 MHz bandwidth, 30 MHz i.f. amplifier and detector. The 1 kHz output of the detector was fed back to the transmitter hut, detected and applied to a pen recorder Y input. The X input was taken from the control unit of the turntable on which the reflector, feed and mixer were mounted. The variable attenuator in the transmitter path was used to calibrate the recorded output signal level.

The feed was movable along the reflector axis for focusing and vertically for aligning receiving and transmitting aerials.

Gain was determined experimentally by comparing, at a number of distances, the signal received by the aerial with that received by a standard horn at the same distance. It was also calculated by graphical integration of the E and H plane patterns.

## 5. Experimental results

Radiation patterns were plotted in both E and H planes with two forms of the aerial. Both used the same feed but while one employed the light alloy reflector the other had a glass-fibre-supported silver-plated nylon mesh copy — the mesh covering only the reflecting zones. The patterns for both reflectors showed no significant differences taking into account the fact that the frequency of the transmitter and the distance between feed and reflector could have differed slightly in corresponding measurements.

Figs. 4 and 5 show typical E and H plane patterns with the feed at the focus. The superimposed templet is calculated from 2(1) with $\phi_0 = 4{\cdot}5^\circ$.

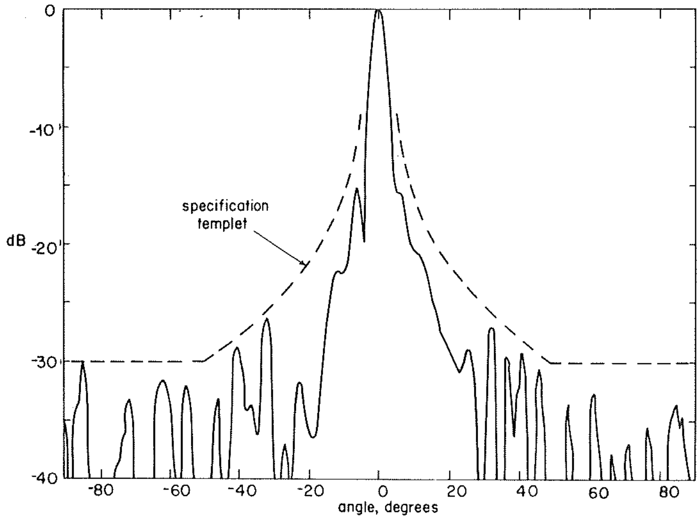

*Fig. 4 - E-plane radiation pattern*

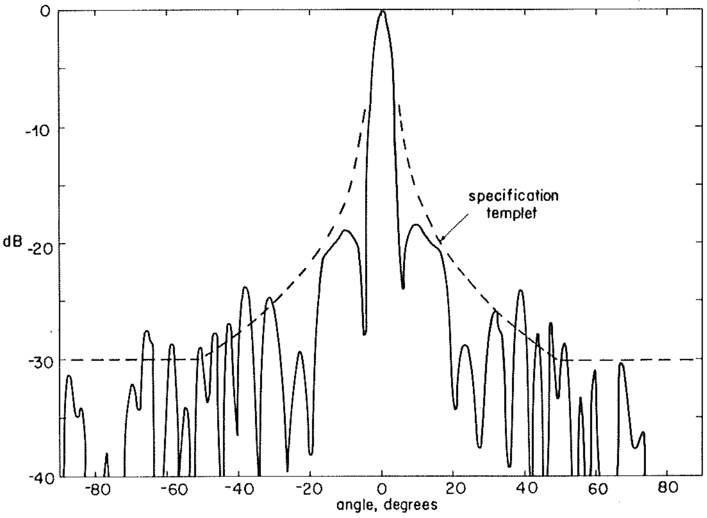

*Fig. 5 - H-plane radiation pattern*

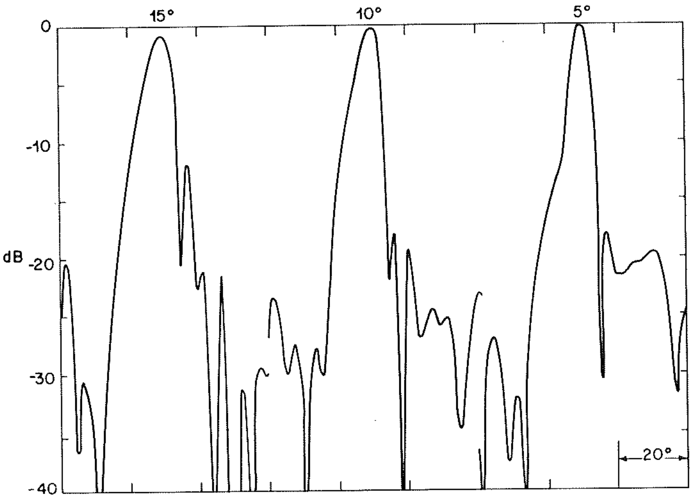

*Fig. 6 - H-plane deflected beam patterns*

Fig. 6 shows the H-plane measured patterns with the feed moved on an approximately circular arc relative to the centre of the reflector. The E-plane patterns are similar with a tendency for the outer side lobes to be lower in amplitude. The patterns for the aerial with the nylon mesh reflector are again very similar.

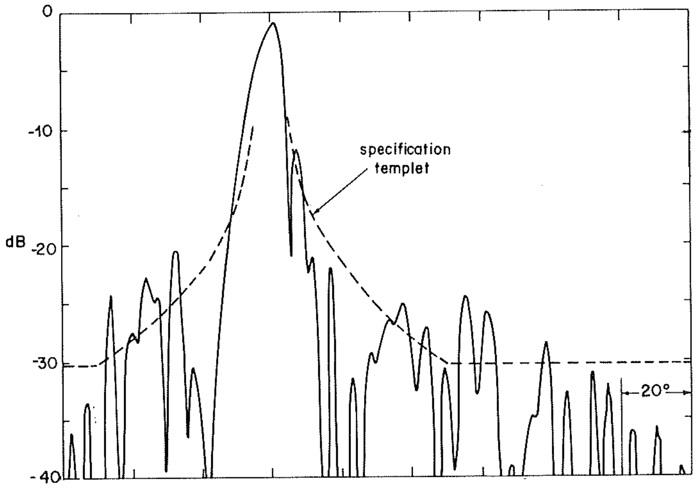

*Fig. 7 - Pattern at 15° beam deflection H-plane*

In Fig. 7 the H-plane radiation pattern for 15° beam scan is compared with the templet calculated from 2(1) with $\phi_0 = 4{\cdot}5^\circ$. This represents a beam scan of 3·3 beamwidths.

Swinging the beam increases the beamwidth and reduces the gain. The results plotted in Fig. 8 show that the gain of the zoned reflector approaches that of a full paraboloidal reflector of the same $f/D$ ratio.[2] In the method used to calculate the radiation pattern of the zoned reflector with the feed at the focus (Appendix II), the feed pattern was approximated on a square law basis and it was assumed that the feed obscured a circular aperture of 5 cm diameter. Fig. 9 shows the calculated response.

## 6. Discussion of results

### 6.1. Radiation pattern

Figs. 4 and 5 show that apart from a few minor transgressions of the templet the aerial patterns meet the off-axis performance specified (2(1)) and this performance is maintained at least over 9225 – 9425 MHz, the range over which measurements were taken.

The calculated pattern (Fig. 9) tends to under-estimate the far side lobes but the allowance made for the feed was not exact, no allowance was made for the feed support and the feed patterns were approximated. Closer attention to these matters could lead to better agreement.

### 6.2. Beamwidth

The 54 cm diameter aerial at $\lambda = 3{\cdot}2$ cms has a total 3 dB beamwidth of 4·5° and on this basis a 75 cm aerial at $\lambda = 2{\cdot}5$ cms would have a beamwidth of 2·5° as proposed in Section 2. Some widening of this beamwidth necessarily occurs when the beam is scanned.

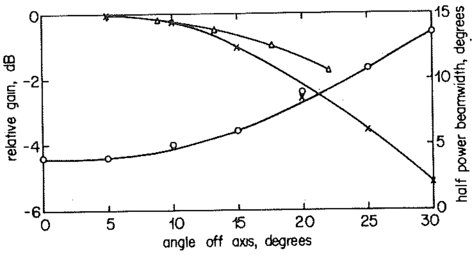

*Fig. 8 - Loss of gain and increase of beamwidth as feed is moved off axis*

o Measured beamwidth (zoned reflector)
x Measured gain (zoned reflector)
Δ Gain of paraboloidal reflector interpolated for same $f/D$ ratio as the zoned reflector from Fig. 13.22, Ref. 2.

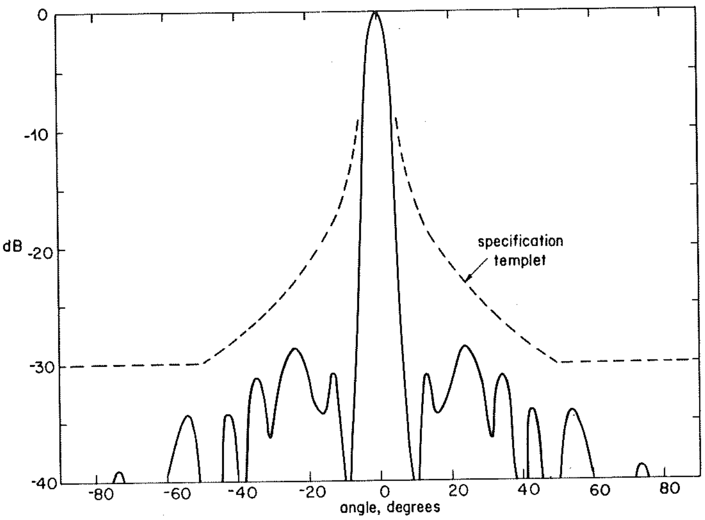

*Fig. 9 - Theoretical radiation pattern*

### 6.3. Gain

The mean gain of the 54 cm diameter aerial at $\lambda = 3{\cdot}2$ cms relative to a 24 dB standard horn measured over a number of distances in the far field was approximately +6 dB which agrees well with the 30·6 dB obtained by integration of the measured E and H plane patterns and the 30·2 dB calculated assuming 50% efficiency of the effective radiating apertures (Fig. 2).

Translated to a 75 cm dish at $\lambda = 2{\cdot}5$ cms this would imply a gain of 35 dB over an isotropic radiator compared with 36·5 dB for a full parabolic dish similarly illuminated.

### 6.4. Scanning

Fig. 6 shows the patterns obtained for 5°, 10° and 15° beam deflection, which are displaced horizontally for clarity. The beam deviation factor (3·2) of the measured patterns approached unity.

The feed horn was moved on an arc of a circle centred at the middle of the reflector. This is not necessarily the optimum path. In the case of some other zoned reflector aerials the best compromise paths were convex to the reflectors.[4,5] These reflectors however were concave to the feed and it is possible that with the flat reflector a traverse of the feed parallel to the reflector may improve the scanning performance, vis-à-vis that of a paraboloid of the same $f/D$ ratio (see Fig. 8).

Fig. 7 shows that at 15° beam deflection (approximately 3·3 beamwidths) there is not excessive transgression of the templet (calculated from 2(1) with $\phi_0 = 4{\cdot}5^\circ$) and only 1 dB loss of gain. For the 75 cm aerial this corresponds to a deflection of 8·3°. With a 75 cm aerial mounted facing due south with a 28·5° elevation, ±5° scan would suffice to cover any site in the United Kingdom (Section 2) and the maximum loss of gain would be of the order of 0·5 dB.

## 7. Suggestions for further study

### 7.1. Mathematical analysis

It is thought that better agreement between calculated and measured radiation patterns would be obtained if closer attention were paid to certain aspects of the calculation as mentioned in Section 6.1. Instead of the approach used a more sophisticated method could be investigated such as a solution in terms of free space modes.[6,7] This could prove useful since so far no attempt has been made to optimise the zones or calculate the scanning performance. This latter would be complicated by reflections from the conical sections. Indeed the performance might be improved by joining the zones by right circular cylindrical surfaces coaxial with the paraboloids or cones having smaller vertex angles.

The optimisation of the feed path in slewing might also be attempted using the paragoemetric approach of Ronchi and Toraldo.[5]

### 7.2. Bandwidth

The performance of the aerial should be checked over a bandwidth corresponding to the percentage bandwidth of Band VI since a stepped reflector is necessarily frequency sensitive and widely spaced broadcasting channels might be allocated to the United Kingdom.

### 7.3. Offset reflector

As an alternative to tilting the whole aerial (6·4) consider Fig. 10. The construction is as described in 3.1 with the tangent plane to the inner-most paraboloid at an angle $\psi$ to the common axis. With the reflector vertical and $\psi = 61{\cdot}5$ the beam will be in the direction VF, that is elevated at an angle $\pi/2 - \psi = 28{\cdot}5^\circ$ which would satisfy the requirement of 6·4 assuming that scanning over ±5° is practicable.

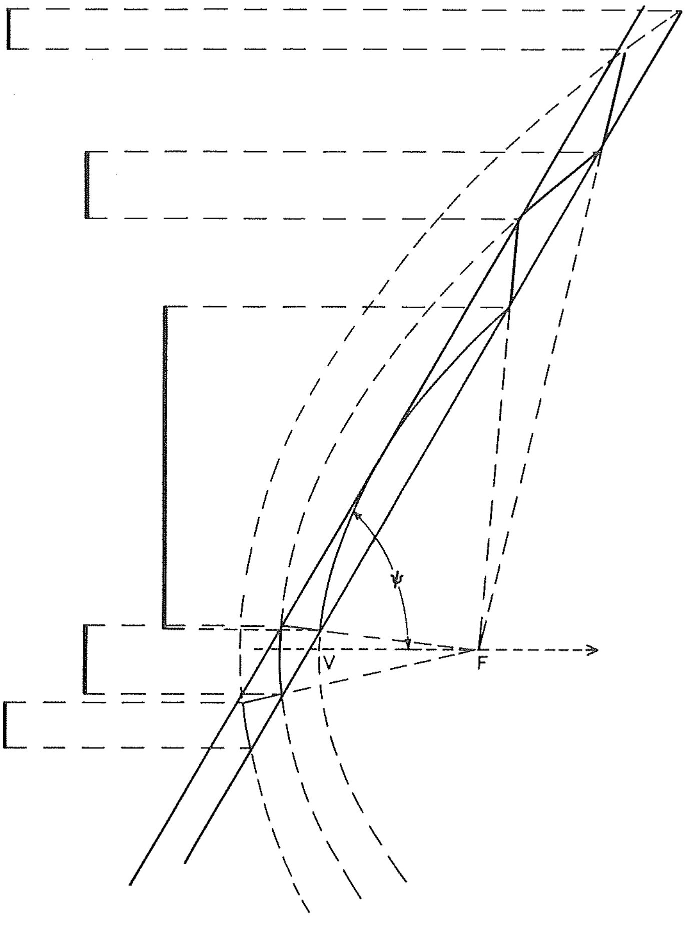

*Fig. 10 - Development of offset zoned reflector*

### 7.4. Wall-mounted aerial

In the case of an aerial mounted on the wall of a house, Fig. 11 shows that the angle between the normal to the wall and the direction of the satellite varies between wide limits, and to cater for walls up to 45° from due south 24° – 55° variation would be required. Some alleviation of this requirement could be obtained by making use of the roof slope, but this could lead to a more expensive installation. This range of angles could be met by a 40° offset and ±15° scanning. With a 75 cm reflector and an $f/D$ approaching 0·5, ±6 beamwidths (i.e. ±15°) could probably be obtained with a symmetrical reflector[2] but further study would be required to determine whether satisfactory performance could be obtained with a 40° offset. It might be more satisfactory to have offsets of 30°, 40° and 50° with ±5° scan or some other combination. The difficulty of machining offset reflectors emphasises the desirability of a mathematical analysis able to predict the aerial performance sufficiently accurately.

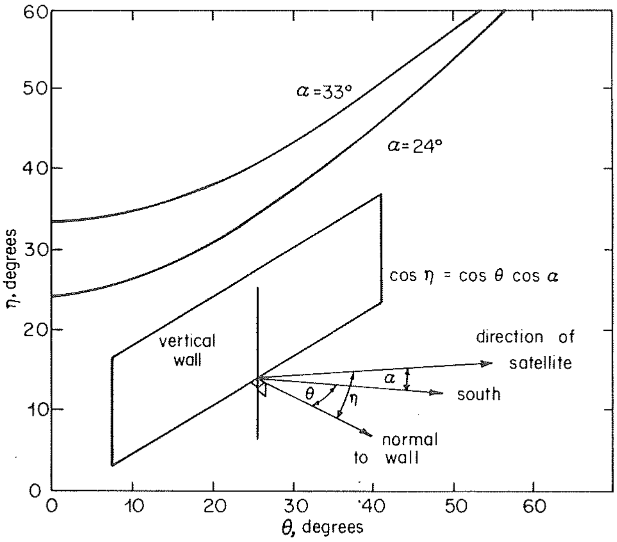

*Fig. 11 - Angle between normal to wall and satellite*

### 7.5. Circular polarisation

Alignment of the receiving aerial with the satellite transmitter using offset and scan (7.4) could require both scanning and rotation of the aerial about the normal to the wall, which would necessitate a circularly polarised aerial to maintain signal level. This would cause a loss of 3 dB in system gain with a plane polarised transmitting aerial, but with potentially interfering transmitters right and left handed circularly polarised, this loss could be avoided and added co-channel protection obtained at the same time.

A waveguide horn could be circularly polarised with little difficulty but compatibility with stripline techniques might more easily be achieved with some form of printed aerial. A printed spiral antenna[8] requires a balanced feed to eliminate bore-sight error[9] and this could be achieved by feeding into a balanced mixer housed, with the local oscillator, immediately behind the radiator. The signals could then be taken down the feed support at UHF or I.F.

The spiral antenna should provide a satisfactory feed since the −10 dB beamwidth of approximately ±60° is only marginally wider than the ±53° of the waveguide horn feed used.

### 7.6. De-focusing

The narrow beamwidth (±1·25°), besides requiring a high degree of stability of the mounting for satisfactory reception, would lead to considerable difficulty in aligning the receiving aerial with the satellite transmitter. This problem could be reduced by defocusing during initial alignment, at the expense of aerial gain, by sliding the feed towards the reflector, trebling the beamwidth perhaps for 10 dB loss of gain.

## 8. Manufacture

Reference has already been made (3.1) to the problem of machining the master reflector. For the offset reflector (7.3) the problem would be somewhat greater and recourse to some form of computer controlled machining would probably be necessary.

A number of methods of making copies of the master reflector are available, e.g. metal pressings, glass-fibre moulding etc. and would have to be investigated before a realistic estimate of the cost of the aerial could be made, but the method adopted in the Research Department, suitably modified for production could produce a fairly inexpensive reflector. In this process male glass-fibre moulds were made from the master reflector and a lay-up consisting of silver-plated nylon mesh backed up by layers of glass-fibre mat was formed on these moulds. The result was a reflector the performance of which, within the limits of the experiment, was indistinguishable from that of the original master.

## 9. Conclusions

An aerial employing a stepped paraboloidal reflector with a diameter of 75 cms could be made substantially to meet the specifications currently assumed for a domestic aerial for the reception of Band VI signals broadcast from a geo-stationary satellite, and scan over ±5° with negligible deterioration of performance.

Mounted facing south at an angle of elevation of 28·5° it would be suitable for any open site in the United Kingdom and would have a gain of 35 dB over an isotropic radiator.

No attempt has been made to assess precise cost but it is believed that in large numbers this aerial could be manufactured relatively cheaply.

## 10. References

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## *APPENDIX I*

### *The Design of a Diffraction Fence*

The purpose of a diffraction fence is to eliminate reflections from the ground between a transmitter and receiver. Consider Fig. 12.

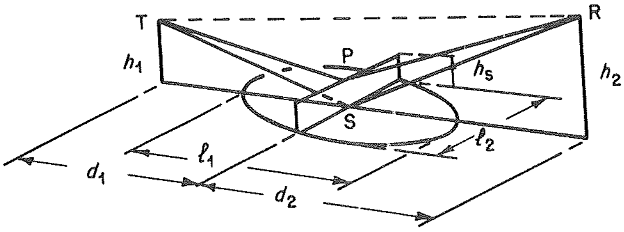

*Fig. 12 - A diffraction fence to eliminate ground reflections*

It is generally recognised[11] that only the area of the ground around the specular reflection point S contributes interfering reflections at the receiver. This area is taken to be the first Fresnel zone in the ground plane, and is an ellipse of major and minor axes $l_1$ and $l_2$, given by

$$l_2 = 2b\sqrt{1 - \frac{c^2}{b^2\cos^2\xi + a^2\sin^2\xi}}$$

$$l_1 = \frac{l_2 a}{\sqrt{b^2\cos^2\xi + a^2\sin^2\xi}}$$

where

$$a = \tfrac{1}{2}\left(d\sec\xi + \frac{2h_1h_2}{d} + \frac{\lambda}{2}\right)$$

$$b = \tfrac{1}{2}\sqrt{2d\sec\xi\left(\frac{2h_1h_2}{d} + \frac{\lambda}{2}\right) + \left(\frac{2h_1h_2}{d} + \frac{\lambda}{2}\right)^2}$$

$$c = \tfrac{1}{2}(h_1 + h_2),\ \xi = \tan^{-1}\left(\frac{h_2 - h_1}{d}\right) \text{ and } d = d_1 + d_2$$

The diffraction fence must obscure the Fresnel zone then; the simplest solution is to erect an opaque vertical screen, height $h_s$, width $l_2$, across the path at S. Height $h_s$ is

$$h_s = \frac{l_1h_1}{2(d_1 + \tfrac{1}{2}l_1)} = \frac{l_1h_2}{2(d_2 + \tfrac{1}{2}l_1)}$$

A second consideration is the effect of the screen on the direct path from transmitter to receiver. This can be assessed from knife-edge diffraction laws. For the configuration given the Fresnel parameter of path TPR is

$$V = -\sqrt{\frac{2d_1d_2}{\lambda(d_1 + d_2)}}\left\{\frac{h_1 - h_s}{d_1} + \frac{h_2 - h_s}{d_2}\right\}$$

in which the term $\{\ \}$, the diffraction angle, is assumed to be small. If the transmitter and receiver have uniform radiation patterns, the diffraction can be assessed as classical Fresnel diffraction; the amplitude of the field received at R as a function of $V$ is shown in Fig. 13.

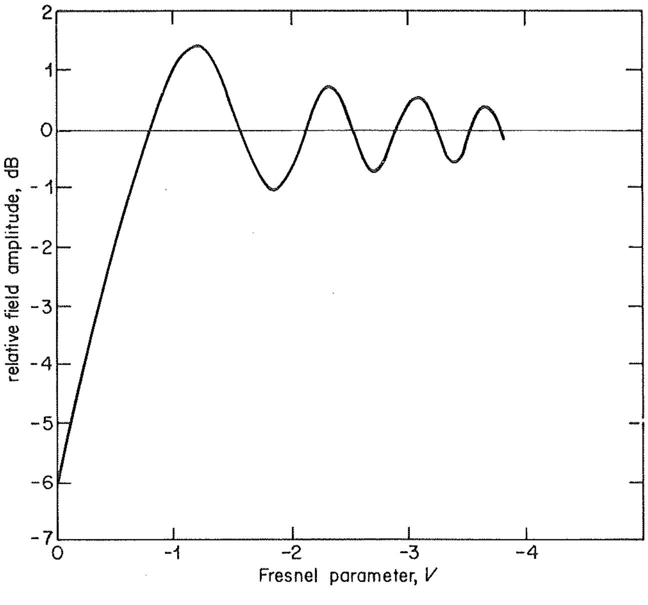

*Fig. 13 - Knife-edge diffraction*

If, on the other hand, the transmitter has a tapered distribution, Budin[12] has shown that the diffraction fringes are reduced in intensity. For $V$ smaller than $-2$, and a uniform beam, the deviation of the received field from the value without the edge present is less than ±1·0 dB. For a tapered beam $V$ may be less negative for the same allowed deviation.

In the present instance; $h_1$ = 1·6 m, $h_2$ = 2·0 m, $d$ = 32 m, $\lambda$ = 32 mm. Then $d_1$ = 14·2 m and $d_2$ = 17·8 m; the Fresnel zone has major and minor axes of length $l_1$ = 8·26 m and $l_2$ = 0·95 m respectively. Thence $h_s$ = 0·38 m, for which height $V = -3{\cdot}92$, well within the limit specified above.

The fence used in the experiments was a metal sheet about 1 m high and 4 m wide, which obscured much more of the ground than just the first Fresnel zone. However, the parameter $V$ was $-2{\cdot}18$, which is still within the deviation limit of ±1·0 dB specified.

## *APPENDIX II*

### *The Radiated Field of a Focally-Fed Stepped Zone Reflector*

On an optical analogy a cone of rays from the focus of a parabola is reflected from the parabolic surface as a cylinder of rays parallel to the axis and apparently originating on the directrix (Fig. 14).

The illumination of the directrix is $E_\theta/r$ where $E_\theta$ is the relative field radiated by the feed at an angle $\theta$[13], and this expressed at a function of $\rho$ was approximated by

$$E = A_n - \rho^2/k_n$$

$A_n$ and $k_n$ corresponding to the n^th^ zone.

Following Silver[14] the contribution of the integral from an annulus $\rho_n < \rho < \rho_{n+1}$ is given by

$$f(\phi) = \int_{\rho_n}^{\rho_{n+1}} (A_n - \rho^2/k_n)\, J_o(k\rho \sin\phi)\rho d\rho$$

where $\phi$ is the azimuth angle.

Putting k sin$\phi$ = a, replacing a$\rho$ by x and using the substitution

$$\frac{d}{dx}\, x^n J_n(x) = x^n J_{n-1}(x)$$

leads to

$$f(\phi) = \frac{1}{a^2} \int_{a\rho_n}^{a\rho_{n+1}} (A_n - x^2/a^2 k_n) d(xJ_1(x))$$

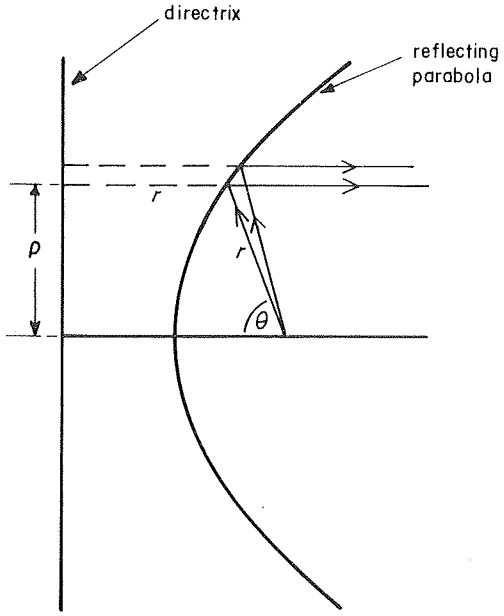

*Fig. 14 - Parabolic reflector*

Integration by parts and a further use of the above substitution finally gives

$$f(\phi) = Q(\rho_{n+1}) - Q(\rho_n)$$

where

$$Q(\rho) = \rho(A_n - \rho^2/k_n)J_1(a\rho)/a + 2\rho^2 J_2(a\rho)/a^2 k_n$$

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Printed by BBC Research Department, Kingswood Warren, Tadworth, Surrey